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A Theoretical Study of Liquid-Film Spread Heights in the Calendering of Newtonian and Power Law Fluids

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Synopsis

The relationship between spread height and upstream reservoir thickness, with power law coefficient as parameter, was obtained analytically. At all values of n studied, the value of r (ratio of spread height to nip width) increases with increasing values of H/h_0 where H is upstream reservoir thickness and h_0 is nip width. At higher values of H/h_0, the curves of r versus H/h_0 tend to "flatten" out, and r approaches an asymptotic value. For example, the asymptotic value of r for Newtonian fluids (power law constant of 1) is 1.226. Asymptotic values of r increase with decreasing values of the power law constant.

INTRODUCTION

Calendering is a widely used method for the production of plastic films. Ability to predict the final thickness (spread height) of the calendered sheet is of obvious importance.

In many industrial applications the fluid located between the rollers is nonuniform in temperature, and the film leaving the rollers is no longer liquid but solid. For the idealized analysis presented in this paper, it will be assumed that the fluid temperature is uniform and that the film leaving the rollers is still liquid.

PREVIOUS WORK

During the past 35 years, a number of papers pertaining to flow between rotating cylinders (calendering) were published, including general reviews of the problem.[1,4,12–14] Most of the papers published during this period have been primarily concerned with deriving theoretical expressions for pressure, shear rate, shear stress, and velocity distributions in the fluid field. However, to calculate the above quantities from the equations presented, the value of spread height has to be known. In general, the authors do not

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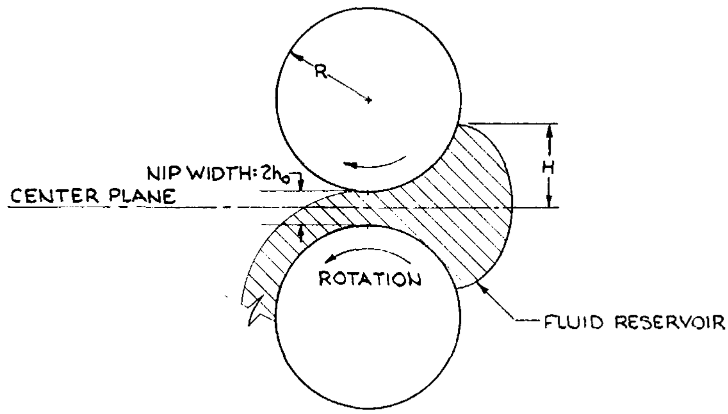

Fig. 1. Ardichvili's assumed model for roller systems. *Note:* Rollers have no translatory motion; radius of rollers (R) much greater than nip width; figure not drawn to scale.

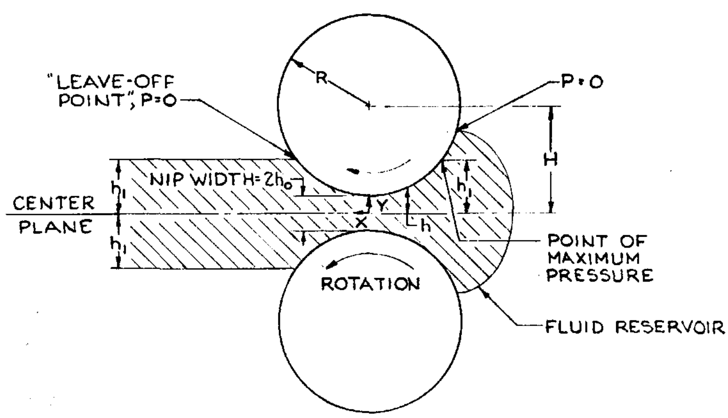

Fig. 2. Gaskell's assumed model for roller systems. *Note:* Rollers have no translatory motion; radius of rollers (R) much greater than nip width; figure not drawn to scale.

present expressions which can be used to calculate values of h_1 (one half of the spread height, and also equivalent to half the distance of separation between the rollers at the point of maximum pressure), nor do they actually present numerical values. The few authors who do discuss spread height generally do not consider the influence of power law constant or upstream reservoir thickness on the value of h_1. They usually present a single value, applicable to a specific situation.

Ardichvili[2] analytically determined that when H (one half the thickness of the upstream fluid reservoir) is much larger than h_0 (one half the nip width), then h_1 is equal to $^4/_3 h_0$. In his analysis, which is restricted to a Newtonian fluid, Ardichvili assumed that the calendered fluid "left off" at the nip and that the final thickness (i.e., spread height) was therefore equal to the nip width ($2h_0$). Thus, in his assumed system, pressure dropped to zero at the nip. (See Fig. 1 for Ardichvili's postulated model and Fig. 2 for a definition of the "leave off" point.) It should be noted that in Ardichvili's analysis, h_1 is equivalent only to one half the distance of separation of the rollers at the point of maximum pressure, and is not equal to one half the spread height.

In calculating pressures and in evaluating thermal effects in the fluid field, Eley[7] also assumed that the fluid "left off" at the nip. In a follow-up evaluation of thermal effects, Finston[9] took a more realistic view of the calendering process. He based his calculations on the model proposed by Gaskell.[10]

Gaskell asserted that in order to analyze the calendering process correctly, it should be assumed that fluid "leaves off" at a certain distance past the nip and that pressure drops to atmospheric at the "leave off" point. The point of maximum pressure is obviously located upstream of the nip (see Fig. 2). Gaskell, however, does not present any explicit expressions for calculating h_1, nor does he present any numerical values.

The validity of Gaskell's assumed model was confirmed by the experimental measurements of Bergen and Scott.[5] These authors demonstrated that the pressure in the nip was quite significant and that the pressure did not drop to ambient (i.e., zero gauge) until some distance past the nip. Bergen and Scott's measured pressure distribution showed much better agreement with calculations based on Gaskell's model than with values computed from Ardichvili's model.

Dexter and Marshall[8] claimed to have used Gaskell's model in analyzing their experimental data but then went ahead and made use of the relation $h_1 = {}^4/_3h_0$. As previously noted, Ardichvili showed that this relationship was true only for a Newtonian fluid under the following two conditions: (1) H is much greater than h_0, and (2) fluid "leaves off" at the nip. Gaskell has shown this second assumption to be incorrect. In Dexter and Marshall's

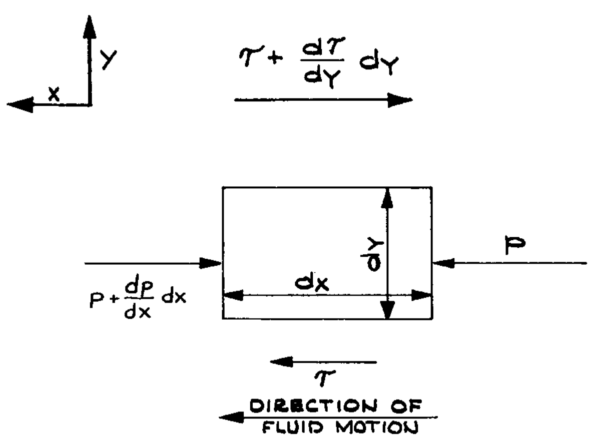

Fig. 3. Forces acting on a fluid element.

experiments, H is much larger than h_0; but since the fluid does not "leave off" at the nip, h_1 is not equal to $^4/_3 h_0$.

None of the above-mentioned authors either calculated or measured values of r as a function of H and n over large ranges of these latter variables. McKelvey[13] calculated r as a function of H/h_0 for a Newtonian fluid, but over a very limited range, i.e., at values of H/h_0 ranging from 1 to 2.

Most of the papers described above assumed that the fluid was Newtonian. However, the more recent papers on the subject have taken pseudoplastic[3,6] and viscoelastic bahavior[6,15,17,18] into account. When the viscoelastic constitutive equations were substituted into the equations of motion, the resulting expressions were very complex and could not be solved analytically. Chong[6] and White and Tokita[17,18] used these complex expressions as a basis for formulating dimensionless groups which are inportant in the calendering process and also in ascertaining which of these groups could be correlated with defects in the calendered sheet.

Chong and White and Tokita did not discuss the factors which determine the value of h_1. Interestingly, though, Atkinson and Nancarrow[3] state that the spread height is equal to the nip thickness unless the fluid has elastic properties. As previously mentioned Gaskell has shown this view to be unrealistic. In addition, Chong's Figure 3[6] implies that the value of r is 1.33, regardless of upstream reservoir thickness or power law constant. The results of the present paper will show that this is also incorrect.

THEORY

Qualitative Aspects

As previously mentioned, the model now generally accepted for a fluid flowing between rotating cylinders is shown in Figure 2. If the fluid were to "leave off" at the nip as Ardichvili proposed, then the pressure would have to fall to ambient at this point. However, the elevated pressure must actually extend past the nip, even in the absence of elastic recovery (viscoelasticity), because the average fluid velocity at the nip is greater than the velocity of the roll surfaces.[10] The fluid will then occupy the space between the rollers until its pressure drops to ambient. The spread height of the fluid, even for a purely viscous liquid, therefore has to be greater than the nip width.

It is the equation of conservation of mass which dictates that the average velocity of the fluid flowing through the nip be greater than the speed of the rollers. At some point upstream of the nip, i.e., where the pressure is a maximum, the velocity profile is flat and the absolute velocity of the fluid is equal to the roller speed. The decrease in flow cross-sectional area in going from this upstream point to the nip necessitates an increase in the average fluid velocity at the nip.

Derivation of Equation Used to Calculate Spread Height

A force balance on an element of fluid moving with the flowing liquid yields (see Fig. 3)

$$\frac{dp}{dx} = \frac{-dT}{dy}. \tag{1}$$

In deriving eq. (1), it was assumed that the inertial forces are negligible compared with the surface (i.e., viscous) forces. This assertion is tantamount to assuming that the acceleration of the particle can be neglected, an assumption usually made in lubricating and calendering analyses. The constitutive equation for a power law fluid is given simply by

$$T = -K\left|\frac{du}{dy}\right|^{n-1} \cdot \left(\frac{du}{dy}\right). \tag{2}$$

Newtonian fluids are only a special case of the more general power law liquid. For a Newtonian fluid, n is equal to unity and K is equal to the viscosity.

Equation (3) results from differentiating eq. (2) and combining the result with eq. (1):

$$\frac{dp}{dx} = nK\left|\frac{du}{dy}\right|^{n-1} \frac{d^2u}{dy^2}. \tag{3}$$

By integrating eq. (3) twice and utilizing the boundary conditions $du/dy = 0$ at $y = 0$, and $u = U$ at $y = h$, we obtain

$$u = \frac{[(1/K)(dp/dx)]^{1/n}}{1 + 1/n} \cdot (y^{1+1/n} - h^{1+1/n}) + U. \tag{4}$$

Combination of eq. (4) with the equation of continuity,

$$Q = 2\int_0^h u dy = 2Uh_1,$$

yields eq. (5), after algebraic simplification:

$$\left(\frac{K}{2}\right)\left[\frac{(2n + 1)2U}{n}\right]^n \frac{|h - h_1|^{n-1}(h - h_1)}{h^{2n+1}} = \frac{dp}{dx}. \tag{5}$$

Equation (5) was transformed into terms of h only, eq. (7), by making the substitution

$$h = h_0 + \frac{x^2}{2R}. \tag{6}$$

Equation (6) is an approximate geometric relationship, but it is reasonably accurate for typical situations in which fluid is flowing between rotating cylinders. All of the references that mathematically analyze the calendering process make use of this approximate relationship:

$$\sqrt{\frac{R}{2}} \cdot \left(\frac{K}{2}\right) \cdot \left[\frac{(2n+1)2U}{n}\right]^n \cdot \frac{|h-h_1|^{n-1}(h-h_1)}{h^{2n+1}\sqrt{h-h_0}} = \frac{dp}{dh}. \quad (7)$$

Essentially, the integrated form of eq. (7) was used to calculate spread height as a function of H, with power law constant n as parameter. The general method for integrating eq. (7) will be described below. For the Newtonian case, i.e., where $n = 1$, eq. (7) can be solved exactly, and an analytical expression relating h_1 to H can be obtained. This exact integration is presented in the appendix. However, when n is less than 1, or is greater than 1 but not an integer, eq. (7) has to be solved by a graphical trial and error method.

Equation (7) can be solved conveniently with an additional variable transformation:

$$\xi = \frac{x}{\sqrt{2Rh_0}} \quad (8)$$

or

$$\xi = \sqrt{\frac{h}{h_0} - 1}. \quad (9)$$

It should be noted that while h only assumes positive values, x and ξ range from positive to negative values.

By combining eqs. (7) and (9), we obtain

$$\frac{\sqrt{2Rh_0}}{h_0^{n+1}} \cdot \left(\frac{K}{2}\right) \cdot \left[\frac{(2n+1)2U}{n}\right]^n \cdot \frac{|\xi^2-\xi_1^2|^{n-1}(\xi^2-\xi_1^2)}{(\xi^2+1)^{2n+1}} = \frac{dp}{d\xi}. \quad (10)$$

The pressure distribution in the fluid field can be calculated by integrating eq. (10). (As previously mentioned, for all noninteger values of n, the integration would have to be done graphically.) Once the pressure distribution has been calculated, the velocity and shear rate distributions can be calculated from eq. (4) and the derivative of eq. (4), respectively. The determined pressure distribution can also be used to calculate the shear stress distribution from eq. (1).

The limits of integration for eq. (10) are established by noting that pressure equals zero (i.e., zero gauge) at $\xi = \xi_1$, (the leave-off point) and that $p = p$ at $\xi = -\xi$, any point upstream of the nip. The integral of eq. (10) may therefore be expressed as

$$\frac{\sqrt{2Rh_0}}{h_0^{n+1}} \cdot \left(\frac{K}{2}\right) \cdot \left[\frac{(2n+1)2U}{n}\right]^n \int_{\xi_1}^{-\xi} \frac{|\xi^2-\xi_1^2|^{n-1}(\xi^2-\xi_1^2)d\xi}{(\xi^2+1)^{2n+1}} = p. \quad (11)$$

The spread height of the liquid was obtained by noting that $p = 0$ at $\xi = -\xi_H$ (the inlet point). Equation (11) therefore becomes

$$\frac{\sqrt{2Rh_0}}{h_0^{n+1}} \cdot \left(\frac{K}{2}\right) \cdot \left(\frac{(2n+1)2U}{n}\right)^n \int_{\xi_1}^{-\xi_H} \frac{|\xi^2-\xi_1^2|^{n-1}(\xi^2-\xi_1^2)d\xi}{(\xi^2+1)^{2n+1}} = 0. \quad (12)$$

Since the term

$$\frac{\sqrt{2Rh_0}}{h_0^{n+1}} \cdot \left(\frac{K}{2}\right) \cdot \left(\frac{(2n+1)2U}{n}\right)^n$$

is not equal to zero, eq. (12) reduces to

$$\int_{\xi_1}^{-\xi_H} \frac{|\xi^2 - \xi_1^2|^{n-1}(\xi^2 - \xi_1^2)d\xi}{(\xi^2 + 1)^{2n+1}} = 0. \tag{13}$$

The geometrical meaning of eq. (13) is best visualized by dividing the above integral into three parts:

$$\underset{\mathrm{I}}{\int_{\xi_1}^{0} \frac{|\xi^2 - \xi_1^2|^{n-1}(\xi^2 - \xi_1^2)d\xi}{(\xi^2 + 1)^{2n+1}}} + \underset{\mathrm{II}}{\int_{0}^{-\xi_1} \frac{|\xi^2 - \xi_1^2|^{n-1}(\xi^2 - \xi_1^2)d\xi}{(\xi^2 + 1)^{2n+1}}}$$

$$+ \underset{\mathrm{III}}{\int_{-\xi_1}^{-\xi_H} \frac{|\xi^2 - \xi_1^2|^{n-1}(\xi^2 - \xi_1^2)d\xi}{(\xi^2 + 1)^{2n+1}}} = 0. \tag{14}$$

A plot of $f(\xi)$ versus ξ is shown in Figure 4. The plot is, of course, applicable for any particular value of n. The direction of integration is also indicated in the figure. Areas I and II represent positive quantities because the integration is carried out in the negative x direction. (Obviously, areas I

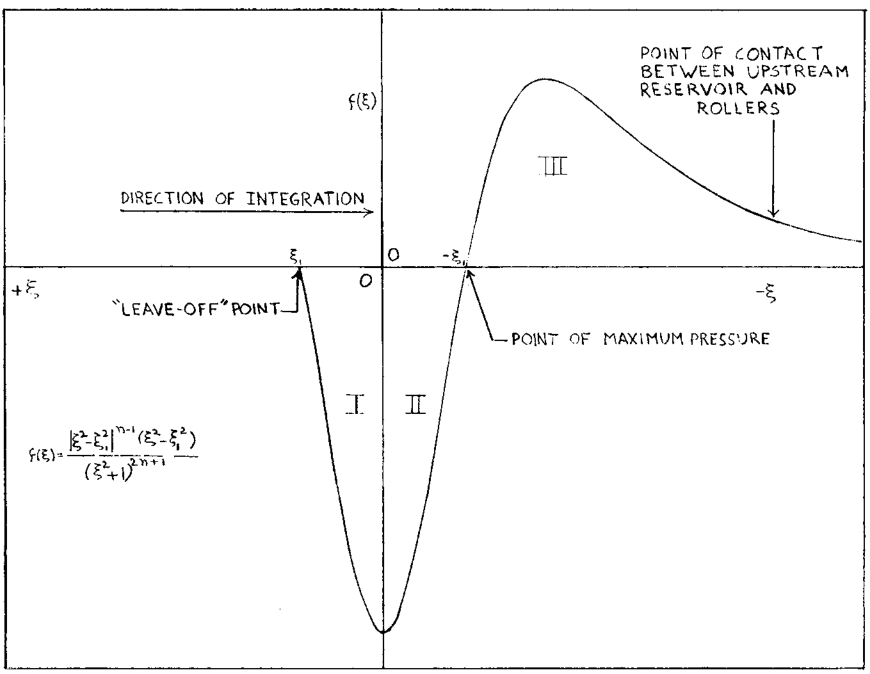

Fig. 4. Plot of $f(\xi)$ versus ξ.

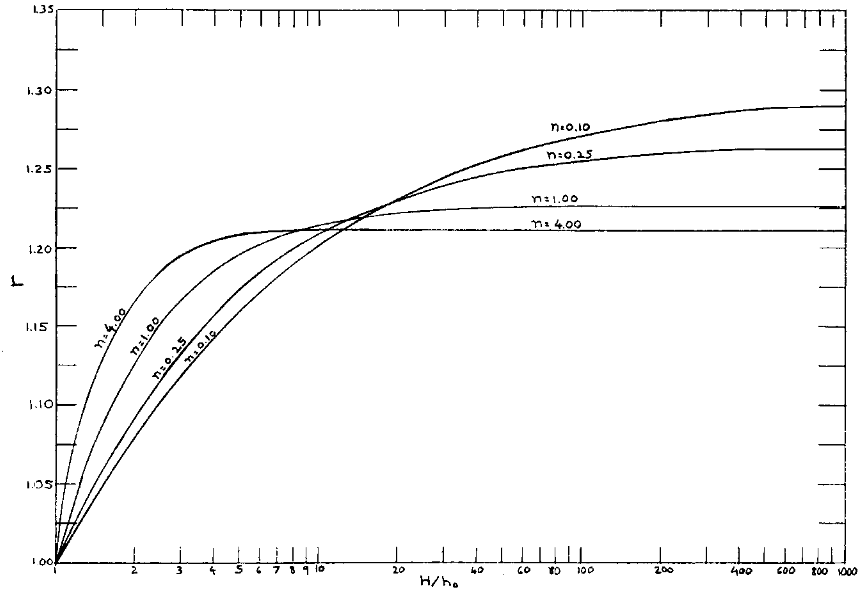

Fig. 5. Spread height as a function of upstream reservoir thickness with power law constant as parameter.

and II are equal.) Area III is negative, and the value of $-\xi_H$ is such that the magnitude of area III is equal to the magnitude of the sums of areas I and II, as dictated by eq. (14).

Solution of Eq. (14) to Determine h_1/h_0 as a Function of H/h_0

The method used to compute h_1/h_0 as a function of H/h_0 for a given value of n is described below. The actual numerical calculations were performed on an IBM 360/30 computer. For reasons of convenience, H is computed as a function of h_1 rather than the reverse. The steps and the order in which they are performed are listed below.

1. A value of ξ_1 is chosen. The value of the integral $\int_{\xi_1}^{0} \{ \quad \} d\xi$ is evaluated graphically using Simpson's rule,[16] and the result is doubled. The final result is equal to the sum of the integrals:

$$\left[\int_{\xi_1}^{0} \{ \quad \} d\xi + \int_{0}^{-\xi_1} \{ \quad \} d\xi.\right]$$

2. A value of ξ_H was then assumed, and the value of the integral

$$\left[\int_{-\xi_1}^{-\xi_H} \{ \quad \} d\xi\right]$$

was compared with

$$\left[\int_{\xi_1}^{0} \{ \quad \} d\xi + \int_{0}^{-\xi_1} \{ \quad \} d\xi.\right]$$

If the values of the above bracketed terms were unequal, a new value of ξ_H was assumed and the value of $\int_{-\xi_1}^{-\xi_H} \{\quad\} d\xi$ was recomputed.

3. The procedure in this step (step 2) was repeated until a value of ξ_H was found such that $\int_{-\xi_1}^{-\xi_H} \{\quad\} d\xi$ was equal to $\int_{\xi_1}^{0} \{\quad\} d\xi + \int_{0}^{-\xi_1} \{\quad\} d\xi$. This value of ξ_H (or H) was taken to be the appropriate value for the chosen value of ξ_1 (or h_1).

4. A new value of h_1 (or ξ_1) was then chosen, and the process (steps 1 through 3) was repeated. In this manner, values of H corresponding to values of h_1, with n as parameter, were obtained (see Fig. 5).

For several different values of h_1, the entire procedure described above (steps 1–4) was repeated using intervals of ξ equal to one quarter of those used in the original calculations. The smaller interval size had no significant effect on the final calculated results.

RESULTS AND DISCUSSION

Curves of r as a function of H/h_0 with the power law constant as parameter are presented in Figure 5. Values of H/h_0 range from 1 to 1000, and curves are presented for values of n equal to 0.1, 0.25, 1.0, and 4.0.

For all values of n, the curve of r versus H/h_0 levels off at "large" values of H/h_0. These "leveled off," or asymptotic, values of r have been labelled r_∞.

It is interesting to note that, at low values of H/h_0, r increases with increasing n. However, at the larger values of upstream reservoir thickness, r increases with decreasing values of n, as can be seen from Figure 5. The

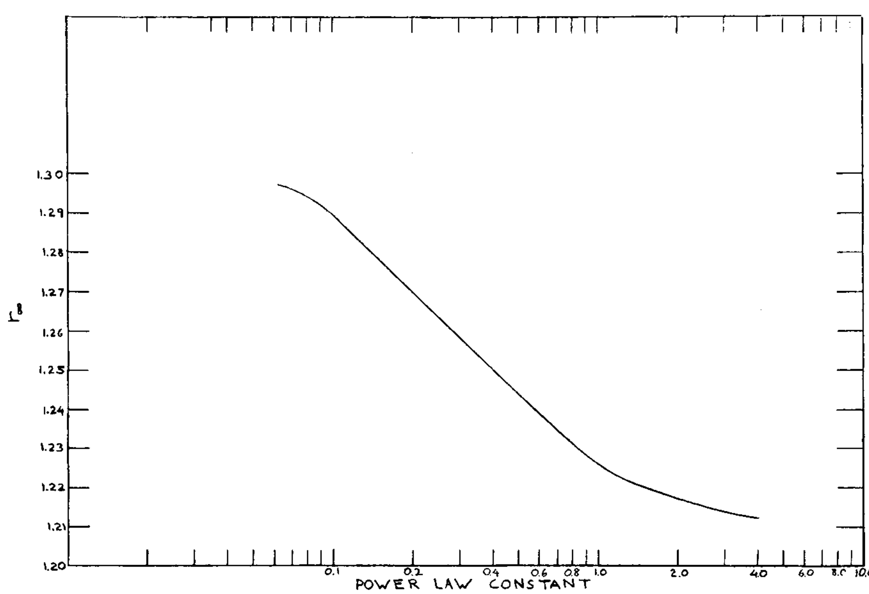

Fig. 6. Asymptotic value of r as a function of power law constant.

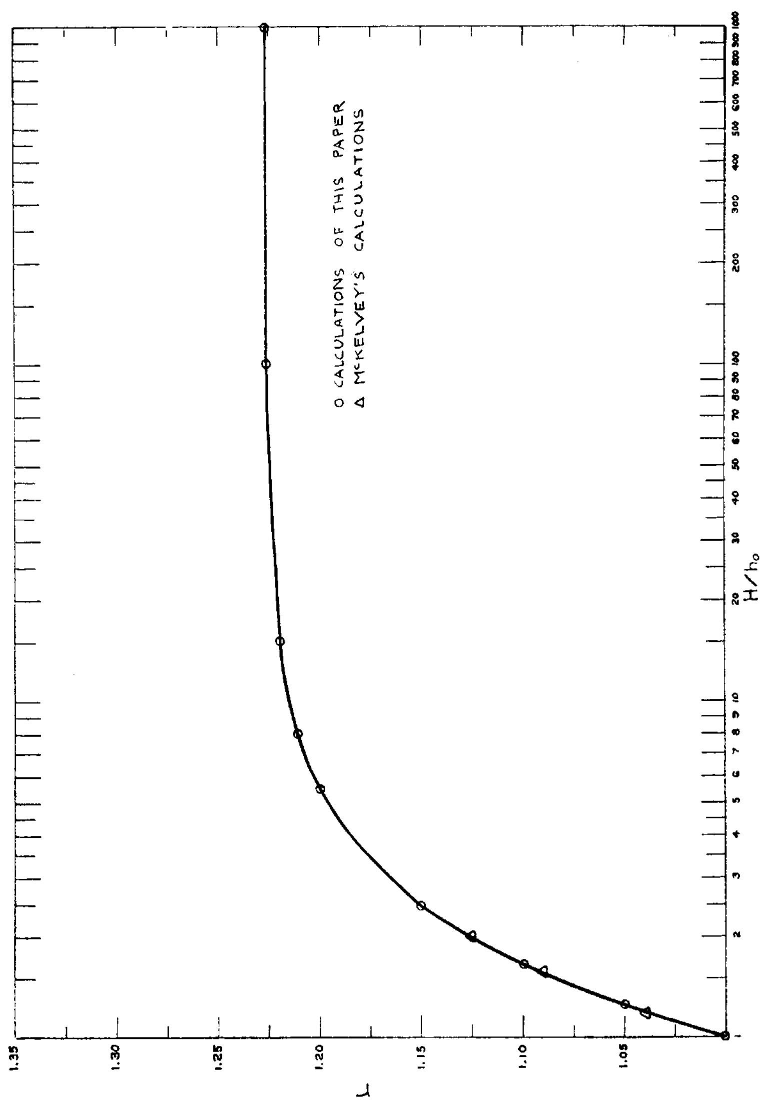

Fig. 7. Comparison of McKelvey's calculations[12] with calculations made in this paper: (O) this paper; (Δ) McKelvey.

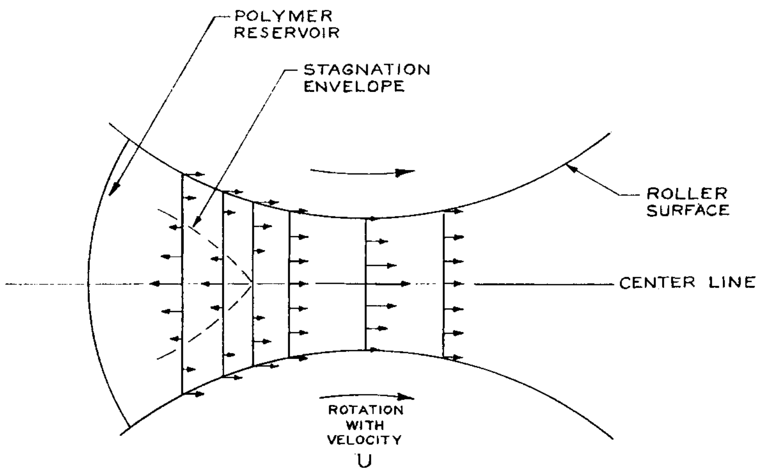

Fig. 8. Velocity distribution between rollers and stagnation envelope. Arrows indicate velocity vectors.

results of the present work also predict that r is independent of roller speed, roller gap, the radius of the rollers, and viscosity (for non-Newtonian fluids it is independent of the constant K).

McKelvey[13] presents values of r at various values of H/h_0 for a Newtonian fluid. His values, along with those calculated in the present paper, are shown in Figure 7. The agreement is excellent. McKelvey's values, however, do not extend to large ratios of H/h_0. Jones[11] calculated the value of r for a Newtonian fluid at a value of H/h_0 equal to infinity. His calculated value of 1.226 is in perfect agreement with the graphically calculated value of the present work.

As previously mentioned, theory predicts that larger values of H result in greater spread heights. Bergen and Scott[5] experimentally confirmed this relationship. An explanation for this phenomenon is as follows: Pressure is ambient at the point of contact between the upstream reservoir and the rollers; it increases continuously to the point of maximum pressure and then decreases to about one half its maximum value when the nip is reached. Pressures are, therefore, higher in the nip at the larger H's because of the greater distance over which pressure can build up. The elevated pressure in the nip has to be "discharged," i.e., reduced to zero gauge, before the fluid can leave the rollers. Since at the higher H's greater pressures in the nip have to be discharged, greater distances from the nip are required to discharge these higher pressures. Thus, from purely qualitative reasoning it is certainly reasonable to expect h_1 to increase with increasing H.

It is interesting to note that the equations described earlier predict the existence of a circulatory motion in the upstream fluid reservoir. In fact, Ancker[7] photographed this motion. Chong[6] actually showed how eqs. (3) and (4) could be used to predict the existence of this circulatory motion in the reservoir. The explanation is as follows: At some position upstream of the nip, the velocity is zero along the centerline. At each position upstream of this centerline stagnation point, there exist two stagnation points (see Fig. 8). The fluid within the envelope formed by the loci of the stagnation points is in backflow, whereas the liquid outside the envelope is flowing in a downstream direction toward the nip. The backflow, coupled with the normal downstream flow, leads to circulatory fluid patterns in the reservoir.

CONCLUSIONS

1. At low values of H/h_0, the ratio r decreases with decreasing n, whereas at the larger values of H/h_0, r increases with decreasing power law constant.
2. At all values of n studied, r approaches an asymptotic value.
3. The functional relationship between r and H/h_0 is independent of roller speed, roller radius, and nip width. It is also independent of viscosity in the case of Newtonian fluids and is independent of K (the constant in the shear rate–shear stress relation) for power law fluids.

APPENDIX

Exact Solution of Newtonian Case

As with the power law case, the derivation of the final equation for a Newtonian fluid can start with eq. (7). Noting that, for Newtonian fluids, $n = 1$ and $K = \mu$, eq. (7) is transformed into

$$dp = 3\mu U \sqrt{\frac{R}{2}}\left[\left(\frac{1}{h^2} - \frac{h_1}{h^3}\right)\frac{1}{\sqrt{h - h_0}}\cdot dh\right]. \qquad \text{(A-1)}$$

Integrating eq. (A-1) from $p = 0$ at $h = h_1$ to $p = p$ at $h = h$, we obtain

$$\int_0^p dp = p = 3\mu U \sqrt{\frac{R}{2}}\left[\int_{h_1}^{h_0}\left(\frac{1}{h^2} - \frac{h_1}{h^3}\right) \times \left(\frac{1}{\sqrt{h - h_0}}\right)\cdot dh - \left(\int_{h_0}^{h}\left(\frac{1}{h^2} - \frac{h_1}{h^3}\right)\frac{dh}{\sqrt{h - h_0}}\right)\right]. \qquad \text{(A-2)}$$

As shown above, the integral was divided into two parts, and a negative sign was placed before the second part. It should be noted that ξ or x can range from positive to negative values, whereas h can only take on positive values. Thus, the negative sign before the second integral in eq. (A-2) is necessary if the entire integral in this equation is to have the same meaning as the integral in eq. (11).

The functional dependence of h_1 on H can be obtained by actually integrating eq. (A-2) and noting that p again equals zero (i.e., zero gauge) when $h = H$. The result is

$$\frac{2r\sqrt{\frac{H}{h_0} - 1}}{(H/h_0)^2} - (4 - 3r)\left(\frac{\sqrt{\frac{H}{h_0} - 1}}{H/h_0} + \tan^{-1}\sqrt{\frac{H}{h_0} - 1}\right) = -\frac{2\sqrt{r - 1}}{r} + (4 - 3r)\left[\frac{\sqrt{r - 1}}{r} + \tan^{-1}\sqrt{r - 1}\right]. \qquad \text{(A-3)}$$

TABLE A-I

H/h_0	$r = h_1/h_0$
2	1.128
5	1.197
10	1.215
101	1.226
∞	1.226

Equation (A-3) was solved by trial and error. The results are listed in Table A-I. As expected, these values are in almost exact agreement with those calculated graphically for the case of $n = 1$.

A quick glance at eq. (A-3) would tend to indicate that for a given value of H/h_0 there are an infinite number of values of r. This apparently would be due to the fact that the arctangent function is multivalued for a given value of its argument. However, in order to solve eq. (A-3) by a method which is mathematically valid, one has to either restrict the value of the arctangent function to principal values (i.e., between $-\pi/2$ and $+\pi/2$), or restrict the values of $\tan^{-1}\sqrt{(H/h_0) - 1}$ and $\tan^{-1}\sqrt{r - 1}$ to the same quadrant. Thus, for every value of H/h_0 there exists a unique value of r.

Nomenclature

h = distance from center plane to periphery of roller at any value of x
h_0 = $^1/_2$ of the minimum gap between rollers, minimum gap also termed nip width
h_1 = $^1/_2$ of the spread height; also equivalent to half the distance of separation between the rollers at the point of maximum pressure
H = $^1/_2$ of the upstream reservoir thickness
K = constant in shear stress–shear rate relationship
n = power law constant
p = pressure
Q = volumetric flow rate
R = radius of rollers
r = ratio of spread height to nip width
r_∞ = asymptotic value of r
u = point or local velocity
U = average velocity
x = position in lateral direction
y = position in logitudinal direction
μ = viscosity
ξ = defined by eq. (9)
ξ_1 = value of ξ corresponding to the "leave off" point; $-\xi_1$, corresponds to point of maximum pressure
$-\xi_H$ = value of ξ corresponding to the point of contact between the upstream reservoir and the rollers
T = shear stress
$\{\ \}$ = symbol for the term $\dfrac{|\xi^2 - \xi_1^2|^{n-1}(\xi^2 - \xi_1^2)}{(\xi^2 + 1)^{2n+1}}$

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